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Consumption and the timing of income risk

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Abstract

When formulating a plan for savings, does the timing of income risks matter? Should one be concerned with whether uncertainty in income is in the near term, or the distant future? Of course, and the impact of the timing of income risk is the subject of this article. Using a three-period framework, we provide approximate solutions for optimal consumption choices for preferences that display constant relative risk aversion and derive the relationship between innovations to income and innovations to consumption growth. These results are contrasted with those for quadratic preferences and preferences that display constant absolute risk aversion. We analyze consumption–saving plans for several different situations of near term and distant future income risk, and different kinds of preferences. We conclude with a demonstration of the high degree of accuracy of our consumption approximations by comparing them to exact values computed by stochastic simulation. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

A classical problem in the theory of intertemporal allocation is the characterization of the connection between income and consumption. The most basic treatment centers on the reaction of current consumption to a change in current income, with the familiar debate concerning whether additional income is immediately spent, or whether the effect is mollified due to planning in a life cycle or permanent income format. These concerns have stimulated an enormous amount of research in the past two decades, that is devoted to formalizing the process of intertemporal planning under uncertainty. Deaton (1993) provides a good recent exposition of this work.

A main focus of consumption theory has been the analysis of precautionary saving, or forward planning that sets money aside in case of bad luck in future income. While the motive for precautionary saving is clear, in any practical situation of intertemporal planning the timing of uncertainty is of particular importance. Different considerations arise for precautionary savings for situations of near term risk, such as a low salary or a temporary layoff from work, versus distant risk, such as concerns in the level of pension benefits in retirement. But what is really at issue is how the full consumption plan is structured with regard to the configuration of future uncertainty, or the entire future evolution of income. Consider, for instance, the savings plan of a thirty-five year old worker who has a five year old daughter. In fifteen years, the worker plans on paying his daughter's college expenses, and in twenty-five years, he plans to retire. Now, imagine that the worker is faced with a change in his pension plan – namely his current and near-term income expectations are not altered, but the characteristics of his income in retirement are changed. How should he alter his saving for retirement now if his pension income becomes more certain, or alternatively, if it becomes very uncertain because of problematic management of his company's pension fund? How should he alter his saving for college expenditures, which are themselves substantially uncertain? And how should he adjust his overall saving plan if the pension changes are, in fact, associated with additional risks in near term salary levels? These are all questions of how a consumption–saving plan are affected by income uncertainty at different points of time in the future.[1]

These concerns do not raise any new issues for the *theory* of intertemporal consumption. It is easy to write out the intertemporal optimization problem, that accounts for the sequential resolution of future uncertainties in income. However, very complicated analytical problems arise when one attempts to get an explicit solution for the optimal consumption plan. This occurs because the

[1] We are studying the reaction to uninsurable risks, and do not study how different our results would be if incomplete insurance markets existed for all future contingencies.

optimal plan depends explicitly on how income uncertainty is revealed over time, and the structure of risk tolerance built into preferences. Current precautionary savings must take into account precautionary savings planned for future periods, and the whole plan must evolve as the future uncertainties are resolved.

In fact, the only explicit solutions known exhibit very unrealistic behavioral implications. Quadratic preferences (cf. Campbell, 1987) yield a solution where saving is done in anticipation of declining income, but with no accommodation for risk. Preferences with *Constant Absolute Risk Aversion* (CARA) yield a solution that accommodates income risk (cf. Caballero, 1990, 1991), but with the level of precautionary savings not varying with overall wealth.[2] Neither of these cases serve to capture reasonable features of precautionary behavior – for instance, the notion that precaution is less necessary if you are, in fact, extremely wealthy. As a reaction to these deficiencies, Skinner (1988), Kimball (1990a) and Carroll (1994) study optimal consumption with preferences displaying *Constant Relative Risk Aversion* (CRRA), where precautionary savings vary inversely with the level of initial wealth.[3] Here, an explicit solution is not available, with the Skinner–Kimball–Carroll work studying approximations to optimal consumption in a two-period framework.

In this paper, we analyze the connection between optimal consumption and the timing of income risk for consumers with CRRA preferences. We focus on the timing features by studying optimal consumption over three periods – now, the near-term future and the distant future. We establish a new approximation to the optimal consumption plan, that captures the timing and sequential revelation of uncertainty in future income. We use our approximation to illustrate how precautionary savings plans vary with different prospects for income uncertainty. We also indicate how our approximation is very accurate within a wide range of potential scenarios.

Two features of our analysis are noteworthy, relative to alternatives. The first is that our approximation captures the explicit connection between current consumption and distant future risk, with the distribution of uncertain future income values summarized by the sequence of wealth normalized variances. The alternative to using our general approximation is stochastic simulation, wherein one postulates a specific distribution for future income innovations, and then solves for optimal consumption numerically. It is clear that stochastic

[2] We cover these cases in detail in Section 2.

[3] Following Kimball (1990a), Dynan (1993) examines how the importance of risk for precautionary saving is proportional to absolute 'prudence', or the negative of the ratio of third to second derivatives of utility. For CARA preferences the proportionality factor is constant whereas for CRRA preferences it is inversely related to the level of consumption – or wealth. However, in these studies, risk terms remain defined in terms of the second derivative of utility and are not explicitly related to the characteristics of income risk.

simulation results only reveal features of consumption in the assumed scenarios, and as such, purely computational approaches cannot usefully separate the influences of near term and distant income risk for a wide range of situations. We have opted to develop an approximation that depends on relative variances, because we feel it gives a more useful depiction of how risk is accounted for in general.

The second feature concerns our focus on three periods, which we have adopted as the simplest setting wherein the timing of risk can be studied. As we will see, there are many more considerations in a three-period setting than a two-period setting. For approaches based on stochastic simulation, modeling four or more periods is associated with much greater numerical complexity for any given scenario, which further obscures the connection between current consumption and distant income risk. In contrast, our approximation is designed to connect the sequential consumption decisions as uncertainty unfolds over time, and is easily extended to many periods. As such, while we focus on optimal consumption plans over three periods, there are no obstacles to using our methodology to study the timing of uncertainty in more general settings. We are not aware of any other solution approach that successfully combines the sequential decisions of an optimal consumption–saving plan.[4]

We begin in Section 2 by introducing the three-period framework and detailing the consumer problem. In Section 3 we review results under perfect certainty and derive approximate solutions for maximization under uncertainty for consumers with CRRA preferences. From these results it is clear how the timing of income risk affects optimal consumption levels, and how the structure of risk affects consumption growth and innovations over time. In Section 4, we apply our results to calculate different consumption paths for different preference structures and different situations of income risk over time. We are able to develop a clear depiction of how risk at different future times is reflected in current consumption, and how that connection is affected by the structure of risk tolerance in preferences. Finally, we illustrate the high degree of accuracy of our approximation to optimal consumption. In particular, we assume precise distributions for income innovations in some of our income risk scenarios, compute the (exactly) optimal consumption plans for those scenarios, and compare them to the consumption values given by our approximation. Section 5 gives some concluding remarks.

[4] There are topics in consumption theory that we do not address, which have been successfully studied with stochastic simulation methods. Liquidity constraints and asset limits for welfare recipients provide two examples. See Deaton (1991) and Hubbard et al. (1995), among others.

2. The three-period model with income risk

2.1. The wealth constraint

We study the choice of consumption expenditures c_0, c_1, c_2 by a consumer over three time periods, indexed by $t = 0, 1, 2$. The budget constraint for intertemporal allocation over the three periods is

$$c_0 + \frac{c_1}{1+r_1} + \frac{c_2}{(1+r_1)(1+r_2)} = W + \frac{\zeta_1}{1+r_1} + \frac{\zeta_2}{(1+r_1)(1+r_2)}. \tag{2.1}$$

Here W is expected wealth at period 0, which contains initial assets as well as the present value of expected income to be received over the three periods. The interest rates r_1 and r_2 are taken to be known for planning purposes.[5] Innovations in income are represented by ζ_1 and ζ_2. In particular, ζ_1 and ζ_2 are unknown as of period 0, ζ_1 is revealed in period 1, and ζ_2 is revealed in period 2. In terms of the introductory remarks, one can think of period 0 as the current time, period 1 as the near-term future and period 2 as the more distant future. Further interpretation can be attached to this set-up; such a life cycle with early worklife (period 0), mid-career (period 1) and retirement (period 2).

From the vantage point of period 0, the income innovations are jointly distributed with zero means: $\mathrm{E}_0(\zeta_1) = 0$ and $\mathrm{E}_0(\zeta_2) = 0$. Define

$$\zeta_2^* = \zeta_2 - \mathrm{E}_1(\zeta_2 \mid \zeta_1), \tag{2.2}$$

where $\mathrm{E}_1(\zeta_2 \mid \zeta_1)$ is the conditional expectation of ζ_2 given ζ_1, so that ζ_2^* has mean 0 from the vantage point of period 1. Further define

$$\zeta_1^* = \zeta_1 + \frac{\mathrm{E}_1(\zeta_2 \mid \zeta_1)}{1+r_2}. \tag{2.3}$$

In words, ζ_1^* and ζ_2^* are the uncorrelated increments to wealth realized in periods 1 and 2, respectively – we can rewrite available wealth in terms of these uncorrelated increments so that

$$c_0 + \frac{c_1}{1+r_1} + \frac{c_2}{(1+r_1)(1+r_2)} = W + \frac{\zeta_1^*}{1+r_1} + \frac{\zeta_2^*}{(1+r_1)(1+r_2)}. \tag{2.4}$$

[5] We regard interest rates as certain for simplicity – as in Skinner (1988), many of the features of our analysis could be generalized to allow for interest rate uncertainty.

We make the further simplifying assumption that the conditional variance of ζ_2^* does not vary with ζ_1^*,[6] and denote the variances of ζ_1^* and ζ_2^* as

$$\Sigma_1^2 = \mathrm{Var}(\zeta_1^*),$$

$$\Sigma_{2|1}^2 = \mathrm{Var}(\zeta_2^* \mid \zeta_1).$$

This notation gives a compact representation of the features required for our study of the three-period allocation. For clarity, it is useful to step back and relate the notation to initial assets and received income. Suppose the consumer begins with initial assets A_0 and will receive income y_0, y_1 and y_2 over the three periods. The present value of expected wealth is then

$$W = A_0 + y_0 + \frac{\mathrm{E}_0(y_1)}{1+r_1} + \frac{\mathrm{E}_0(y_2)}{(1+r_1)(1+r_2)}.$$

The income innovations ζ_1 and ζ_2 are just

$$\zeta_1 = y_1 - \mathrm{E}_0(y_1),$$

$$\zeta_2 = y_2 - \mathrm{E}_0(y_2),$$

or the forecast errors in future income from the vantage point of period 0. Now, when y_1 (and therefore ζ_1) is learned in period 1, the expected value of y_2 is revised to

$$\mathrm{E}_0(y_2) + \mathrm{E}_1(\zeta_2 \mid \zeta_1)$$

with a remaining forecast error

$$\zeta_2^* = y_2 - \mathrm{E}_0(y_2) - \mathrm{E}_1(\zeta_2 \mid \zeta_1).$$

In other words, in period 1, expected wealth is incremented by ζ_1 plus the revision of the forecast of y_2; namely by ζ_1^*. A natural situation would occur if increments to income were to partially cumulate over time; we will occasionally adopt the specification

$$\mathrm{E}_1(\zeta_2 \mid \zeta_1) = b\zeta_1, \tag{2.5}$$

where b gauges the amount of the income innovation ζ_1 that is repeated in period 2.[7]

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[6] This assumption removes a source of (additional) nonlinearity from our optimal consumption calculations. We will note how relaxing it can be accommodated.

[7] See Blundell and Preston (1998). Clearly if income y obeys a random walk, then $b = 1$. Further, our framework may depart from various multiplicative income growth schemes in the conditional variance assumption. For instance, suppose $y_2 = y_1\eta_2$ where η_2 is independent of ζ_1 with mean 1. Then $\zeta_2^* = (\eta_2 - 1)[\mathrm{E}_0(y_1) + \zeta_1]$, and the conditional variance of ζ_2^* varies with ζ_1.

2.2. The consumption problem

The consumer's problem is to choose c_0, c_1 and c_2 to maximize expected utility of the form

$$U_0(c_0) + \frac{1}{1+\delta_1} \mathrm{E}_0[U_1(c_1)] + \frac{1}{(1+\delta_1)(1+\delta_2)} \mathrm{E}_0[U_2(c_2)]$$

subject to the wealth constraint (Eq. (2.1)) and the timing of choices and revelations. The felicity function $U_t(c_t)$ will be specified to accommodate different kinds of risk tolerance below, and is allowed to depend on t to accommodate planned variations in expenditures. The subjective discount rates δ_1 and δ_2 are taken as known. We denote the ratios of market to subjective discount factors as

$$\phi_1 = \frac{1+\delta_1}{1+r_1}, \quad \phi_2 = \frac{1+\delta_2}{1+r_2} \tag{2.6}$$

where the standard case of equal rates coincides with $\phi_1 = \phi_2 = 1$.

We solve for the optimal solution by backward induction. This formally involves solving three problems, namely for c_2, c_1 and then c_0 – but the first is trivial. In particular, given optimal choices for c_0 and c_1, c_2 is given from Eqs. (2.1) and (2.4) as

$$\begin{aligned} c_2 &= (1+r_1)(1+r_2)(W-c_0) - (1+r_2)c_1 + (1+r_2)\zeta_1^* + \zeta_2^* \\ &= (1+r_2)(W_1 - c_1) + \zeta_2^* \end{aligned} \tag{2.7}$$

where W_1 denotes the available wealth in period 1, or

$$W_1 \equiv W_1(c_0, \zeta_1^*) = (1+r_1)(W-c_0) + \zeta_1^*. \tag{2.8}$$

The next step, referred to as "Problem 1" below, is to solve for c_1 given c_0 and the optimal choice c_2 above. In particular, we choose c_1 to maximize

$$U_1(c_1) + \frac{1}{(1+\delta_2)} \mathrm{E}_1[U_2((1+r_2)(W_1 - c_1) + \zeta_2^*)] \tag{2.9}$$

where E_1 is the expectation over ζ_2^* conditional on ζ_1. The first-order condition (or Euler condition) for this maximization is

$$\mathrm{E}_1[U_2'((1+r_2)(W_1 - c_1) + \zeta_2^*)] = \phi_2 U_1'(c_1) \tag{2.10}$$

where we note from above that $(1+r_2)(W_1 - c_1) + \zeta_2^* = c_2$ and $\phi_2 = (1+\delta_2)/(1+r_2)$. Denote the solution to Problem 1 as $c_1(c_0, \zeta_1^*)$.

The last step, referred to as "Problem 0", is to solve for c_0 given the optimal choice $c_1(c_0, \zeta_1^*)$. c_0 is chosen to maximize

$$U_0(c_0) + \frac{1}{1+\delta_1} \mathrm{E}_0[U_1(c_1(c_0, \zeta_1^*))] + \frac{1}{(1+\delta_1)(1+\delta_2)} \times \mathrm{E}_0[U_2((1+r_2)[(1+r_1)(W-c_0) - c_1(c_0,\zeta_1^*) + \zeta_1^*] + \zeta_2^*)] \quad (2.11)$$

where E_0 refers to expectation over ζ_1^* and ζ_2^*.

Much of our analysis involves comparing the optimal consumption plan under uncertainty to the plan that would be optimal under perfect certainty; or when the income innovations vanish; $\zeta_1^* = 0$ and $\zeta_2^* = 0$ with probability 1. We denote the perfect certainty solutions with superscript "o"– $c_0^{\mathrm{o}}, c_1^{\mathrm{o}}, c_2^{\mathrm{o}}$ maximize

$$U_0(c_0) + \frac{1}{1+\delta_1} U_1(c_1) + \frac{1}{(1+\delta_1)(1+\delta_2)} U_2(c_2) \quad (2.12)$$

subject to

$$c_0 + \frac{c_1}{1+r_1} + \frac{c_2}{(1+r_1)(1+r_2)} = W.$$

Likewise, we denote $W_1^{\mathrm{o}} \equiv (1+r_1)(W-c_0^{\mathrm{o}})$, etc. From Eq. (2.11) it can be seen that the entire future pattern of income risk will, in general, influence the optimal level of consumption.

We now turn to computing optimal levels of consumption and consumption growth. Our main focus is on the case of the isoelastic CRRA preference structure. These provide important differences to the more standard analyses that use Quadratic and CARA preferences. However, Quadratic and CARA preferences provide a useful point of comparison and will be helpful in Section 4 when we assess the importance of specific income risk scenarios on the timing of consumption. For quadratic preferences, although an explicit solution is available, income risk plays no direct role; only the means of unknown income innovations enter optimal consumption values (see the Appendix for derivations).

For CARA preferences an explicit solution is also available. CARA preferences are characterized by an exponential felicity function

$$U_t(c_t) = \frac{\mathrm{e}^{\theta(\alpha_t - c_t)}}{-\theta}, \quad (2.13)$$

where α_t reflects planned variations in expenditures over time, and θ reflects the degree of risk aversion.[8] These preferences give rise to a simplistic adjustment

[8] Caballero (1990) gives a detailed analysis of consumption with CARA preferences.

for income risk; more risk gives rise to additive adjustments in c_0 and c_1 that are independent of the level of wealth (see the Appendix for derivations). For instance, suppose that c_1 were optimally lowered by \$500 as a precaution for risk in period 2. Then \$500 is the optimal adjustment whether available wealth W_1 is \$10,000 or 100 million dollars. This feature makes CARA preferences particularly inflexible, and inappropriate for empirical analysis. Even if the parameters of the CARA model were set to give a plausible value of precautionary savings in one period, in a multiperiod framework in which subsequent periods wealth is revised, the original parameter choice is unlikely to give plausible results.

3. Risk adjustments and constant relative risk aversion

Constant relative risk aversion implies risk adjustments that vary with the level of consumer wealth,[9] in a fashion that we view as much more realistic than the solutions for quadratic or CARA preferences. We focus on optimal allocations for the three period setting with (isoelastic) CRRA preferences. Unfortunately, it is not possible to derive explicit consumption and saving solutions with CRRA preferences, and so we derive approximations to the optimal solutions. We use the approximations to study the basic issues of timing of income risk and precautionary savings.

3.1. CRRA preferences and perfect certainty

CRRA preferences are characterized by the felicity function

$$\begin{aligned} U_t(c_t) &= \alpha_t\left(\frac{c_t^{1+\lambda}}{1+\lambda}\right), \quad \lambda < 0, \lambda \neq -1, \\ &= \alpha_t \ln(c_t), \qquad \lambda = -1, \end{aligned}$$

with marginal utility given as

$$U_t'(c_t) = \alpha_t c_t^{\lambda}.$$

The elasticity of intertemporal substitution is constant and equal to $-1/\lambda$, with the log-case associated with unitary elasticity of substitution.

When there is perfect certainty (no income risk), CRRA preferences give a particularly interpretable solution for optimal consumption; namely fixed shares of wealth are allocated to consumption in each period. We now review

[9] See Skinner (1988), Kimball (1990b), Carroll (1994) and Zeldes (1989), among others.

this feature in order to facilitate our general multiperiod analysis. The log case with $U_t(c_t) = \alpha_t \ln(c_t)$ is a natural starting point, with intertemporal preferences (2.12) in the familiar Cobb–Douglas form

$$\alpha_0 \ln(c_0) + \frac{\alpha_1}{1+\delta_1} \ln(c_1) + \frac{\alpha_2}{(1+\delta_1)(1+\delta_2)} \ln(c_2) \tag{3.1}$$

where, without loss of generality, we normalize α_0, α_1 and α_2 so that

$$\alpha_0 + \frac{\alpha_1}{1+\delta_1} + \frac{\alpha_2}{(1+\delta_1)(1+\delta_2)} = 1.$$

Maximizing preferences with subject to the wealth constraint with $\zeta_1 = \zeta_2 = 0$ gives consumption in each period as a fixed share of initial wealth:

$$c_0^o = \alpha_0 W,$$

$$c_1^o = \alpha_1 \left(\frac{1+r_1}{1+\delta_1}\right) W = \frac{\alpha_1}{\phi_1} W,$$

$$c_2^o = \alpha_2 \left(\frac{1+r_1}{1+\delta_1}\right)\left(\frac{1+r_2}{1+\delta_2}\right) W = \frac{\alpha_2}{\phi_1 \phi_2} W.$$

From the vantage point of period 1, the available wealth is

$$W_1^o = (1+r_1)(W - c_0^o) = (1+r_1)(1-\alpha_0)W.$$

and the optimal period 0 and 1 consumption choices are equivalently expressed as

$$c_0^o = \omega_0^o W, \quad c_1^o = \omega_1^o W_1^o \tag{3.2}$$

where the shares are given as

$$\omega_0^o = \alpha_0 \quad \text{and} \quad \omega_1^0 = \frac{\alpha_1}{\alpha_1 + \dfrac{\alpha_2}{1+\delta_2}}.$$

The general CRRA case with $U_t(c_t) = \alpha_t (c_t^{1+\lambda}/(1+\lambda))$ also has a fixed share solution under perfect certainty. Now, with perfect certainty ($\zeta_1 = \zeta_2 = 0$), maximization in period 1 gives the solutions

$$c_1^o = \omega_1^o W_1^o \quad \text{and} \quad c_2^o = (1-\omega_1^o)(1+r_2) W_1^o$$

where

$$\omega_1^o = \frac{\Theta_1^{1/\lambda}}{\alpha_1^{1/\lambda} + \Theta_1^{1/\lambda}}, \quad \Theta_1 = \frac{\alpha_2 (1+r_2)^{1+\lambda}}{1+\delta_2}, \tag{3.3}$$

and $W_1^o = (1 + r_1)(W - c_0^o)$ is the available wealth in period 1 (and c_0^o was chosen in period 0). The optimal value for c_0 from maximization in period 0 is

$$c_0^o = \left[\frac{(\Theta_{01} + \Theta_{02})^{1/\lambda}}{\alpha_0^{1/\lambda} + (\Theta_{01} + \Theta_{02})^{1/\lambda}} \right] W \equiv \omega_0^o W \tag{3.4}$$

where

$$\Theta_{01} = \frac{\alpha_1 (\omega_1^o)^{1+\lambda} (1 + r_1)^{1+\lambda}}{1 + \delta_1},$$

$$\Theta_{02} = \frac{\alpha_2 (1 - \omega_1^o)^{1+\lambda} (1 + r_1)^{1+\lambda} (1 + r_2)^{1+\lambda}}{(1 + \delta_1)(1 + \delta_2)}. \tag{3.5}$$

Thus, we have more complicated formulae, but a fixed share scheme of allocating wealth to consumption over the time periods.

Our approach for solving the problem of allocation under uncertainty is to employ approximations that retain the fixed share structure of the perfect certainty solutions. In particular, we develop approximations of the optimal solutions under uncertainty that take the form

$$c_0 = \omega_0^* W, \quad c_1 = \omega_1^* W_1 \tag{3.6}$$

where, as above, W is initial wealth and W_1 is the available wealth in period 1. The factors ω_0^* and ω_1^* are (approximate) shares that depend on preference parameters as well as the profile of income risk over the three periods. For instance, if income risk increases in period 2, the share ω_1^* decreases, as consistent with precautionary savings. This structure is summarized behaviorally as follows: the consumer will spend the share ω_0^* of his wealth now, spend ω_1^* of his realized (and remaining) wealth next period, and then spend whatever is left in period 2.

We now turn to a derivation of the approximation Eq. (3.6). We then cover the implications of the approximation for consumption growth in Section 3.3. Following this, in Section 4 we give numerical results that address many issues of consumption and the timing of income risk, as well as demonstrate the high degree of accuracy exhibited by the approximation Eq. (3.6) in several scenarios. Readers not interested in the specifics of the derivation may skip to these sections.[10]

[10] The specific formulae defining the shares ω_0^* and ω_1^* are given in Eqs. (3.11), (3.13), (3.18), (3.19) and (3.20), for the logarithmic case, and Eqs. (3.21) and (3.23) for the general case.

3.2. Derivation of the approximate solutions for CRRA preferences

We begin as before with logarithmic preferences (3.1). For Problem 1 the consumer chooses c_1 to maximize

$$\alpha_1 \ln(c_1) + \frac{\alpha_2}{1+\delta_2} \mathrm{E}_1\{\ln[(W_1 - c_1)(1+r_2) + \zeta_2^*]\}.$$

The first-order condition for this optimization is

$$\frac{\alpha_1}{c_1} = \frac{\alpha_2}{1+\delta_2} \mathrm{E}_1\left[\frac{1}{(W_1 - c_1) + \dfrac{\zeta_2^*}{1+r_2}}\right]. \tag{3.7}$$

Rewrite this slightly as

$$\frac{\alpha_1}{c_1} = \frac{\alpha_2}{1+\delta_2}\left(\frac{1}{W_1 - c_1}\right) \mathrm{E}_1\left[\frac{(W_1 - c_1)}{(W_1 - c_1) + \dfrac{\zeta_2^*}{1+r_2}}\right]. \tag{3.8}$$

Aside from the $\mathrm{E}_1[.]$ term, this is the first-order condition for choosing c_1 in the perfect certainty case, so our aim will be to approximate the $\mathrm{E}_1[.]$ term to exploit this connection. Notice first, however, that $\mathrm{E}_1[.] > 1$ by Jensen's inequality (unless $\zeta_2^* = 0$), so the equilibrating level of marginal utility is higher under uncertainty, and the optimal value of c_1 is lower (i.e. the precautionary savings motive).

We approximate the $\mathrm{E}_1[.]$ term by a second-order Taylor expansion of the integrand around the perfect certainty values. We first highlight the importance of scaling the income risk terms by wealth and rewrite the integrand in terms of percentage wealth variations from period 0. Namely, with

$$\Omega_2 \equiv \frac{W_1 - c_1}{W(1+r_1)},$$

$$\rho_2 \equiv \frac{\zeta_2^*}{W(1+r_1)(1+r_2)}$$

we have that

$$\mathrm{E}_1\left[\frac{(W_1 - c_1)}{(W_1 - c_1) + \dfrac{\zeta_2^*}{1+r_2}}\right] = \mathrm{E}_1\left[\frac{\Omega_2}{\Omega_2 + \rho_2}\right].$$

By evaluating the second-order expansion[11] around (Ω, ρ) at the perfect certainty values $(\Omega, \rho) = (\Omega_2^{\mathrm{o}}, 0)$ and then integrating, we note that all terms but the first and last vanish,[12] and we conclude that

$$\mathrm{E}_1\left[\frac{\Omega_2}{\Omega_2 + \rho_2}\right] \cong 1 + \frac{1}{(\Omega_2^{\mathrm{o}})^2}\,\sigma_{2|1}^2$$

where the conditional risk term $\sigma_{2|1}^2$ represents income risk relative to expected wealth and is given by

$$\sigma_{2|1}^2 \equiv \mathrm{Var}(\rho_2) = \frac{\Sigma_{2|1}^2}{W^2(1 + r_1)^2(1 + r_2)^2}. \tag{3.9}$$

Since Ω_2 is the percentage of initial wealth allocated to period 2, under perfect certainty we have

$$\Omega_2^{\mathrm{o}} = \frac{\alpha_2}{(1 + \delta_1)(1 + \delta_2)}$$

so that our approximation to the expectation is

$$\mathrm{E}_1\left[\frac{\Omega_2}{\Omega_2 + \rho_2}\right] \cong 1 + \frac{(1 + \delta_1)^2(1 + \delta_2)^2}{\alpha_2^2}\,\sigma_{2|1}^2. \tag{3.10}$$

Now, if we define

$$\alpha_2^* = \alpha_2\left(1 + \frac{(1 + \delta_1)^2(1 + \delta_2)^2}{\alpha_2^2}\,\sigma_{2|1}^2\right) \equiv \alpha_2 + \beta_{22}\sigma_{2|1}^2, \tag{3.11}$$

we can approximate the first-order condition (3.8) as

$$\frac{\alpha_1}{c_1} = \frac{\alpha_2^*}{1 + \delta_2}\left(\frac{1}{W_1 - c_1}\right). \tag{3.12}$$

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[11] The second-order expansion of the integrand around the point (Ω, ρ) is

$$\begin{aligned}\frac{\Omega_2}{\Omega_2 + \rho_2} &= \frac{\Omega}{(\Omega + \rho)} + \left[\frac{1}{(\Omega + \rho)} - \frac{\Omega}{(\Omega + \rho)^2}\right](\Omega_2 - \Omega) + \left[-\frac{\Omega}{(\Omega + \rho)^2}\right](\rho_2 - \rho)\\ &+ \frac{1}{2}\left[-\frac{2}{(\Omega + \rho)^2} + \frac{2\Omega}{(\Omega + \rho)^3}\right](\Omega_2 - \Omega)^2 + \left[-\frac{1}{(\Omega + \rho)^2} + \frac{2\Omega}{(\Omega + \rho)^3}\right](\Omega_2 - \Omega)(\rho_2 - \rho)\\ &+ \frac{1}{2}\left[\frac{2\Omega}{(\Omega + \rho)^3}\right](\rho_2 - \rho)^2 + rem\end{aligned}$$

where the remainder *rem* is comprised of third- and higher-order terms.

[12] It is interesting to note that if we were to use a higher-order Taylor expansion, the coefficient of $(\Omega_2 - \Omega)^j$ for any j will vanish when evaluated at $\rho = 0$, since $\Omega_2/(\Omega_2 + \rho_2) = 1 - \rho_2/(\Omega_2 + \rho_2)$.

This has the form of the first-order condition for the perfect certainty case except for the variance-adjusted weight α_2^* which replaces α_2. Using $W_1 \equiv (1+r_1)(W-c_0)+\zeta_1^*$ from Eq. (2.8) we can solve for c_1:

$$\mathrm{c}_1 = \omega_1^* W_1 = \omega_1^*[(1+r_1)(W-c_0)+\zeta_1^*] \tag{3.13}$$

where

$$\omega_1^* = \frac{\alpha_1}{\alpha_1 + \dfrac{\alpha_2^*}{1+\delta_2}}.$$

Finally, we are able to write

$$\begin{aligned} c_2 &= (1-\omega_1^*)(1+r_2)W_1 + \zeta_2^* \\ &= (1-\omega_1^*)(1+r_2)[(1+r_1)(W-c_0)+\zeta_1^*]+\zeta_2^*. \end{aligned} \tag{3.14}$$

Therefore, under our approximation, in period 1 the consumer adjusts for risk by spending the fraction ω_1^* of available wealth, instead of the larger fraction ω_1^{o}.

In period 0, the consumer chooses c_0 to maximize

$$\begin{aligned} &\alpha_0 \ln c_0 + \mathrm{E}_0\left[\frac{\alpha_1}{1+\delta_1}\ln(c_1) + \frac{\alpha_2}{(1+\delta_1)(1+\delta_2)}\ln(c_2)\right] \\ &\quad \cong \alpha_0 \ln c_0 + \mathrm{E}_0\left[\frac{\alpha_1}{1+\delta_1}\ln(\omega_1^*[(1+r_1)(W-c_0)+\zeta_1^*])\right] \\ &\qquad + \mathrm{E}_0\left[\frac{\alpha_2}{(1+\delta_1)(1+\delta_2)}\ln[(1-\omega_1^*)(1+r_2)[(1+r_1)(W-c_0)\right. \\ &\qquad \left. + \zeta_1^*]+\zeta_2^*)]\right]. \end{aligned}$$

The first-order condition for this maximization is

$$\begin{aligned} \frac{\alpha_0}{c_0} &= \frac{\alpha_1}{1+\delta_1}\left(\frac{1}{W-c_0}\right)\mathrm{E}_0\left[\frac{(W-c_0)}{(W-c_0)+\dfrac{\zeta_1^*}{1+r_1}}\right] \\ &\quad + \frac{\alpha_2}{(1+\delta_1)(1+\delta_2)}\left(\frac{1}{(W-c_0)}\right) \\ &\quad \times \mathrm{E}_0\left[\frac{(W-c_0)}{(W-c_0)+\dfrac{\zeta_1^*}{1+r_1}+\dfrac{\zeta_2^*}{(1-\omega_1^*)(1+r_1)(1+r_2)}}\right]. \end{aligned} \tag{3.15}$$

Again, by Jensen's inequality, the presence of uncertainty will raise the approximate equilibrating level of marginal utility, leading to a lower value of c_0.

Now apply the same kind of approximation to each of the $\mathrm{E}_0[.]$ terms. First, to write everything in terms of percentages of present values of wealth, we define

$$\Omega_1 = (W - c_0)/W,$$

$$\rho_1 = \zeta_1^*/W(1 + r_1),$$

and we retain $\rho_2 = \zeta_2^*/W(1 + r_1)(1 + r_2)$ from before. The second-order expansion gives

$$\mathrm{E}_0\left[\frac{(W - c_0)}{(W - c_0) + \dfrac{\zeta_1^*}{1 + r_1}}\right] = \mathrm{E}_0\left[\frac{\Omega_1}{\Omega_1 + \rho_1}\right] \cong 1 + \frac{1}{(\Omega_1^{\circ})^2}\sigma_1^2$$

and

$$\mathrm{E}_0\left[\frac{(W - c_0)}{(W - c_0) + \dfrac{\zeta_1^*}{1 + r_1} + \dfrac{\zeta_2^*}{(1 - \omega_1^*)\,(1 + r_1)\,(1 + r_2)}}\right]$$

$$= \mathrm{E}_0\left[\frac{\Omega_1}{\Omega_1 + \rho_1 + \dfrac{\rho_2}{(1 - \omega_1^*)}}\right] \cong 1 + \frac{1}{(\Omega_1^{\circ})^2}\left(\sigma_1^2 + \frac{\sigma_{2|1}^2}{(1 - \omega_1^*)^2}\right) \tag{3.16}$$

where[13]

$$\sigma_1^2 \equiv \mathrm{Var}(\rho_1) = \frac{\Sigma_1^2}{W^2(1 + r_1)^2}\,. \tag{3.17}$$

Since Ω_1 is the fraction of wealth allocated to periods 1 and 2, we have

$$\Omega_1^{\circ} = 1 - \alpha_0.$$

Finally, notice that if we set

$$\alpha_1^{**} = \alpha_1 + \frac{\alpha_1}{(1 - \alpha_0)^2}\,\sigma_1^2 \equiv \alpha_1 + \beta_{11}\sigma_1^2 \tag{3.18}$$

[13] As mentioned in Section 2, some models of income growth will have the variance of ζ_2^* depending on ζ_1^* as $\sigma_{2|1}^2(\zeta_1^*)$ which would imply that ω_1^* depends on ζ_1^* in our solution. The only difference this would make is that $\sigma_{2|1}^2/(1 - \omega_1^*)^2$ in Eq. (3.16) is replaced by $\mathrm{E}_1[\sigma_{2|1}^2(\zeta_1^*)/(1 - \omega_1^*(\zeta_1^*))^2]$, which would need to be evaluated or approximated directly.

and

$$\alpha_2^{**} = \alpha_2 + \frac{\alpha_2}{(1-\alpha_0)^2}\left(\sigma_1^2 + \frac{\sigma_{2|1}^2}{(1-\omega_1^*)^2}\right)$$
$$\equiv \alpha_2 + \beta_{21}\left(\sigma_1^2 + \frac{\sigma_{2|1}^2}{(1-\omega_1^*)^2}\right), \tag{3.19}$$

then we have approximated the first-order condition (3.15) by

$$\frac{\alpha_0}{c_0} = \frac{\alpha_1^{**}}{1+\delta_1}\left(\frac{1}{W-c_0}\right) + \frac{\alpha_2^{**}}{(1+\delta_1)(1+\delta_2)}\left(\frac{1}{W-c_0}\right),$$

which is solved by

$$c_0 = \left(\frac{\alpha_0}{1 + \frac{\alpha_1^{**}-\alpha_1}{1+\delta_1} + \frac{\alpha_2^{**}-\alpha_2}{(1+\delta_1)(1+\delta_2)}}\right)W \equiv \omega_0^* W. \tag{3.20}$$

This gives the approximate solution (3.6) for the logarithmic case.

The general CRRA case with $U_t(c_t) = \alpha_t(c_t^{1+\lambda}/(1+\lambda))$ in treated similarly. We give the specific formulae below, with detailed steps for all derivations provided in Blundell and Stoker (1997). Under uncertainty, the first-order condition (2.10) for Problem 1 is

$$\alpha_1 c_1^\lambda = \frac{\alpha_2(1+r_2)}{1+\delta_2}\,\mathrm{E}_1[(1+r_2)(W_1-c_1)+\zeta_2^*]^\lambda$$
$$= \frac{\alpha_2(1+r_2)^{1+\lambda}}{1+\delta_2}(W_1-c_1)\mathrm{E}_1\left[\frac{(1+r_2)(W_1-c_1)+\zeta_2^*}{(1+r_2)(W_1-c_1)}\right]^\lambda.$$

and our approach is to approximate the $\mathrm{E}_1[.]$ term as before. Denoting

$$\beta_{22}^\lambda = \frac{\lambda(\lambda-1)}{2(1-\omega_0^{\mathrm{o}})^2(1-\omega_1^{\mathrm{o}})^2}$$

and

$$\Theta_1^* = \Theta_1(1+\beta_{22}^\lambda\sigma_{2|1}^2)$$

(where Θ_1 is given in Eq. (3.3)), this gives the approximate optimal consumption choices

$$c_1 = \omega_1^* W_1 \tag{3.21}$$

and

$$c_2^{\mathrm{o}} = (1-\omega_1^*)(1+r_2)W_1 + \zeta_2^* \tag{3.22}$$

where

$$\omega_1^* = \frac{\Theta_1^{*\,1/\lambda}}{\alpha_1^{1/\lambda} + \Theta_1^{*\,1/\lambda}}.$$

For Problem 0, we solve for the value of c_0 that maximizes

$$U_0(c_0) + \frac{1}{1+\delta_1}\,\mathrm{E}_0(U_1\{\omega_1^*[(1+r_1)(W-c_0)+\zeta_1^*]\})$$
$$+\frac{1}{(1+\delta_1)(1+\delta_2)}\,\mathrm{E}_0(U_2\{(1-\omega_1^*)\,(1+r_2)[(1+r_1)(W-c_0)$$
$$+\zeta_1^*]+\zeta_2^*\}).$$

Again, by approximating the latter $\mathrm{E}_0[.]$ terms as before, the approximate optimal consumption choice is

$$c_0 = \omega_0^* W \tag{3.23}$$

where[14]

$$\omega_0^* = \frac{\Theta_0^{*\,1/\lambda}}{\alpha_0^{1/\lambda} + \Theta_0^{*\,1/\lambda}}\,.$$

Again, we have more complicated formulae than in the logarithmic case, but the same fixed share features. One easy comparison allows us to see the impact of the substitution elasticity on risk adjustments. Since $\lambda(\lambda-1)$ appears in the risk adjustment factor β_{22}^{λ} (and β_{11}^{λ}), as expected, larger risk adjustments are associated with larger values of $|\lambda|$, or smaller values of the elasticity of substitution $-1/\lambda$.

3.3. Consumption growth with CRRA preferences

In this section we look specifically at relationship between consumption innovations and income innovations with CRRA preferences. In contrast to the cases of quadratic or CARA preferences, we have that consumption growth

[14] With

$$\beta_{11}^{\lambda} = \frac{\lambda(\lambda-1)}{2(1-\omega_0^{\circ})^2}\,,$$

$$\Theta_0^* = \Theta_{01}^{+}(1+\beta_{11}^{\lambda}\sigma_1^2) + \Theta_{02}^{+}\left[1+\beta_{11}^{\lambda}\left(\sigma_1^2+\frac{\sigma_{2|1}^2}{(1-\omega_1^*)^2}\right)\right],$$

$$\Theta_{01}^{+} = \frac{\alpha_1(\omega_1^*)^{1+\lambda}(1+r_1)^{1+\lambda}}{1+\delta_1},$$

and

$$\Theta_{02}^{+} = \frac{\alpha_2(1-\omega_1^*)^{1+\lambda}(1+r_1)^{1+\lambda}(1+r_2)^{1+\lambda}}{(1+\delta_1)\,(1+\delta_2)}\,.$$

under CRRA will depend on the entire future path of income risk and the responsiveness to income risk depends on the overall wealth level.

The main ideas and structure are available from logarithmic preferences (3.1), so we focus on that case. Begin with the connection between relative income innovations and innovations in marginal utility. For the optimal allocation between periods 1 and 2, the Euler equation is

$$\frac{\alpha_2}{c_2} = \phi_2 \frac{\alpha_1}{c_1}(1+\varepsilon_2)$$

where optimization requires $\mathrm{E}_1(\varepsilon_2) = 0$. Rewrite this as

$$\frac{c_2}{c_1}\frac{\phi_2\alpha_1}{\alpha_2} = \frac{1}{1+\varepsilon_2} \equiv u_2. \tag{3.24}$$

Using Eqs. (3.8) and (3.14) we have

$$\frac{c_2}{c_1}\frac{\phi_2\alpha_1}{\alpha_2} = \left(\frac{(1+r_2)(W_1-c_1)+\zeta_2^*}{(1+r_2)(W_1-c_1)}\right)\mathrm{E}_1\left[\frac{(W_1-c_1)}{(W_1-c_1)+\dfrac{\zeta_2^*}{1+r_2}}\right] \tag{3.25}$$

and letting E_{12} denote the latter expectation, u_2 becomes

$$u_2 = \mathrm{E}_{12} + \frac{\mathrm{E}_{12}}{(1+r_2)(W_1-c_1)}\zeta_2^*. \tag{3.26}$$

This gives a linear relationship between the consumption innovation u_2 and the income innovation ζ_2^*. Moreover, $\mathrm{E}_1(u_2) = \mathrm{E}_{12} \geq 1$, with equality holding only when $\Sigma_{2|1}^2 = 0$, or when $\zeta_2^* = 0$ with probability 1. Expected consumption growth increases with added risk, as E_{12} increases with $\Sigma_{2|1}^2$.

We can develop this relationship more explicitly using the approximation (3.10) and (3.11) resulting in

$$u_2 \cong 1 + \beta_{22}\sigma_{2|1}^2 + \frac{\phi_2\alpha_1}{\alpha_2}\frac{\zeta_2^*}{c_1}. \tag{3.27}$$

Expected growth increases with $\sigma_{2|1}^2$ and is linear in the normalized income innovation ζ_2^*/c_1. Notice that $\sigma_{2|1}^2$ is the variance of the period 2 income innovation scaled by wealth.[15] Adopting the approximation $\ln(u_2) \simeq u_2 - 1$, this can then be applied to consumption growth $\ln c_2 - \ln c_1 = \Delta \ln c_2$ resulting in

$$\Delta \ln c_2 = -\ln\frac{\phi_2\alpha_1}{\alpha_2} + \beta_{22}\sigma_{2|1}^2 + \frac{\phi_2\alpha_1}{\alpha_2}\frac{\zeta_2^*}{c_1}. \tag{3.28}$$

[15] This formulation is used in the empirical study of precautionary saving by Banks et al. (1997).

This is the analog of Kimball's (1990b) two-period consumption growth equation, written in terms of income innovations.

For the optimal allocation between periods 0 and 1, the Euler equation is

$$\frac{\alpha_1}{c_1} = \phi_1 \frac{\alpha_0}{c_0}(1+\varepsilon_1)$$

where again $\mathrm{E}_0(\varepsilon_1)=0$. Again we define the innovation

$$\frac{c_1}{c_0}\frac{\phi_1\alpha_0}{\alpha_1} = \frac{1}{1+\varepsilon_1} \equiv u_1.$$

To derive u_1, we now must appeal to Eq. (3.15) and write

$$\begin{aligned}\frac{c_1}{c_0}\frac{\phi_1\alpha_0}{\alpha_1} &= \omega_1\left(1+\frac{\zeta_1^*}{(1+r_1)(W-c_0)}\right)\mathrm{E}_0\left[\frac{(W-c_0)}{(W-c_0)+\dfrac{\zeta_1^*}{1+r_1}}\right] \\ &\quad + \frac{\omega_1^*\alpha_2}{\alpha_1(1+\delta_2)}\left(1+\frac{\zeta_1^*}{(1+r_1)(W-c_0)}\right) \\ &\quad \times \mathrm{E}_0\left[\frac{(W-c_0)}{(W-c_0)+\dfrac{\zeta_1^*}{1+r_1}+\dfrac{\zeta_2^*}{(1-\omega_1^*)(1+r_1)(1+r_2)}}\right] \qquad (3.29)\end{aligned}$$

where $\omega_1^* = c_1/W_1$, the share at the optimal allocation. Consequently, if E_{01} and E_{02} denote the RHS expectations, we have that

$$\begin{aligned}u_1 &= \omega_1^*\left(\mathrm{E}_{01}+\frac{\alpha_2}{\alpha_1(1+\delta_2)}\mathrm{E}_{02}\right) \\ &\quad + \omega_1^*\left(\mathrm{E}_{01}+\frac{\alpha_2}{\alpha_1(1+\delta_2)}\mathrm{E}_{02}\right)\frac{\zeta_1^*}{(1+r_1)(W-c_0)}. \qquad (3.30)\end{aligned}$$

Expected consumption growth increases with increased σ_1^2, or added risk in period 1. Relation (3.30) gives an exact expression of consumption innovations in terms of income innovations. This is a potentially nonlinear relationship between u_1 and ζ_1^*, because the optimal share ω_1 may vary with ζ_1^*. In part this reflects the richness of the three period design – a large positive ζ_1^* realization that substantially raises wealth will be associated with proportionately smaller precautionary savings.

To develop the implications for consumption innovations, we again appeal to our approximation. Using Eq. (3.20), we have

$$u_1 \cong \frac{(1+\delta_1)\alpha_0}{\alpha_1}\frac{\omega_1^*(1-\omega_0^*)}{\omega_0^*} + \frac{\phi_1\alpha_0\omega_1^*}{\alpha_1}\frac{\zeta_1^*}{c_0}$$

which gives a decomposition of consumption growth into expected growth and the income innovation. It is useful to note that our approximation does not reflect the potential nonlinearity of u_2 in ζ_1^* since the share value ω_1^* does not vary with the realised value of ζ_1^*. We can spell this out in more detail as

$$u_1 \cong 1 + \frac{\omega_1^*}{\alpha_1}\left(\beta_{11}\sigma_1^2 - \frac{\beta_{22}\sigma_{2|1}^2}{1+\delta_2} + \frac{\beta_{21}\left(\sigma_1^2 + \dfrac{\sigma_{2|1}^2}{(1-\omega_1^*)^2}\right)}{1+\delta_2} + \phi_1\alpha_0\frac{\zeta_1^*}{c_0}\right).$$

This expression verifies how expected consumption growth is increasing in σ_1^2. The impact of increasing $\sigma_{2|1}^2$ is ambiguous – we cannot deduce the net effect on u_1 without specifying the parameters. This is related to how the net allocation to period 1 can either increase or decrease in comparison to the perfect certainty case. We illustrate this issue in Section 4.

The general case of CRRA preferences exhibits similar structure, with a traceable impact of the substitution elasticity λ. In particular, solving for consumption growth and employing similar approximations as before we find

$$\ln c_2 - \ln c_1 = \frac{1}{\lambda}\ln\frac{\phi_2\alpha_1}{\alpha_2} - \frac{1}{\lambda}\beta_{22}^{\lambda}\sigma_{2|1}^2 + \frac{\zeta_2^*}{(1+r_2)(W_1 - c_1)} \tag{3.31}$$

which exhibits precisely the same characteristics toward income risk and the income innovation as in the two-period logarithmic case. For initial consumption growth, again we have the same features, as in

$$\ln c_1 - \ln c_0 = -\frac{1}{\lambda}\ln\frac{\Theta_0^*}{\alpha_0} + \ln\frac{\omega_1^*}{1+r_2} + \frac{\zeta_1^*}{(1+r_1)(W - c_0)} \tag{3.32}$$

where Θ_0^* is increasing in σ_1^2 but ambiguous in $\sigma_{2|1}^2$. As before, it is the normalized innovation to income or wealth that enters consumption growth. As uncertainty about the future rises, so will $W - c_0$, dampening the impact of the innovation.

4. Calculating the impact of income risk

One way to understand the differences in the formulations above is to consider the implications for consumption and savings for various scenarios of income risk over time. We design our income scenarios to reflect a typical pattern of income of the life cycle. Income risk profiles are chosen to highlight the importance of the timing of income risk and its severity. We begin by considering the substantive results of the impact of income risk on consumption. After that we show the ability of our approximations to provide an accurate picture of consumption choices over a wide range of scenarios.

Table 1
Perfect certainty and severe retirement uncertainty

	Perfect certainty		Scenario 3.3 log case		Difference
	Consumption	Saving	Consumption	Saving	
c_0	34,305	– 4305	33,741	– 3741	– 563
c_1	34,305	15,695	33,932	16,068	– 373
c_2	34,305	– 14,305	36,816	– 16,816	2511

Note: 'Difference' is the difference in spending from certainty (the negative of precautionary savings).

For several of our comparisons we employ a base case scenario that is intended to capture some of the flavor of planning over a moderate and long time horizon. In particular, we imagine that each of the three periods represents a twenty-year span of life, associated with early career (ages 20–40), mid-career (ages 40–60) and retirement (ages 60–80). We suppose that expected annual income for each of these age classes is \$30,000, \$50,000 and \$20,000 respectively. We suppose that these are after tax incomes, with the retirement income set by a pension plan (that is unaffected by the consumption decisions). While we do not account for discounting of incomes within each period, we discount between periods in a fashion consistent with an annual real interest rate of 3%. That is, we take $(1 + r_1) = (1 + r_2) = (1.03)^{20} = 1.806$, so that the discount factors are $1/(1 + r_1) = 0.554$ and $1/(1 + r_1)(1 + r_2) = 0.307$. This gives a present value of the 60-year income stream of $W = \$1{,}276{,}298$.

We consider CRRA preferences with no variations in planned expenditures, with $\alpha_0 = \alpha_1 = \alpha_2$. We assume that discounting occurs at the market rate of interest, so that $\delta_1 = r_1$ and $\delta_2 = r_2$. We will report the results of difference income specifications in terms of the differences between consumption with income risk and the perfect certainty values. With these specifications, the perfect certainty annual consumption values are the same for all of the CRRA preference specifications; namely they set equal consumption in each period. Table 1 contains the perfect certainty results. We also include in this table the results for logarithmic preferences under one of our uncertainty specifications (Scenario 3.3), as a method of making clear what we report in the tables that follow.

The main focus of the tables are on initial consumption, or c_0 values. These values reflect planning that accommodates all uncertainty in future periods. The c_1 values are included for illustration of the typical consumption plan, but they are based on a realised income innovation of zero in period 1.[16] In particular,

[16] We could compute the appropriate c_1 and c_2 values for any realizations of the income innovations but we just choose zero values for simplicity.

the present values of the c_0, c_1 and c_2 stream equal the present value of the perfect certainty case. The relative size of c_1 and c_2 reflect the effect of the timing of uncertainty on the relative allocation of wealth between periods 1 and 2.

4.1. Effects of income risk and timing

We first consider the optimal consumption plans under seven different scenarios. We focus on the timing of income risk by varying the overall amount of risk and its configuration between periods. Here there is no connection assumed between the income processes of the later periods, so that, for instance, a high-income draw in mid-career does not affect the expected retirement income. We relax this feature in Section 4.3. The results are determined by the variances specified, with the "income ranges" just based on rough normality (namely the mean $\pm$ 2 standard deviations).[17] The scenarios are

1. *Moderate uncertainty*: Here we assess the impact of timing on a moderate amount of income uncertainty and consider two scenarios
   1. *Mid-career*: $\Sigma_1 = \$5000$, $\Sigma_{2|1} = 0$, corresponding to $\sigma_1^2 = 0.001882$ and $\sigma_{2|1}^2 = 0$.
   2. *Retirement*: $\Sigma_1 = \$0$, $\Sigma_{2|1} = \$5000$, corresponding to $\sigma_1^2 = 0$ and $\sigma_{2|1}^2 = 0.000577$.

2. *Severe uncertainty*: Here we assess the impact of timing on a severe amount of income uncertainty and consider the following two scenarios:
   1. *Mid-career*: $\Sigma_1 = \$10{,}000$, $\Sigma_{2|1} = 0$, corresponding to $\sigma_1^2 = 0.007528$ and $\sigma_{2|1}^2 = 0$.
   2. *Retirement*: $\Sigma_1 = \$0$, $\Sigma_{2|1} = \$10{,}000$, corresponding to $\sigma_1^2 = 0$ and $\sigma_{2|1}^2 = 0.003208$.

3. *Mid-career and retirement uncertainty*: Here both mid-career and retirement income are uncertain. We consider three scenarios. In scenario 3.1 the uncertainty is evenly balanced. In scenario 3.2 the near term uncertainty is more severe whereas in scenario 3.3 distant uncertainty is more severe.
   1. *Moderate balanced uncertainty*: $\Sigma_1 = \$5000$, $\Sigma_{2|1} = \$5000$, corresponding to $\sigma_1^2 = 0.001882$ and $\sigma_{2|1}^2 = 0.000577$.
   2. *Mixed mid-career uncertainty*: $\Sigma_1 = \$10{,}000$, $\Sigma_{2|1} = \$5000$, corresponding to $\sigma_1^2 = 0.007528$ and $\sigma_{2|1}^2 = 0.000577$.
   3. *Mixed retirement uncertainty*: $\Sigma_1 = \$5000$, $\Sigma_{2|1} = \$10{,}000$, corresponding to $\sigma_1^2 = 0.001882$ and $\sigma_{2|1}^2 = 0.002308$.

[17] Later we look at an extreme situation of just one high and one low income value, where the appropriate income range would be the mean $\pm$ 1 standard deviation.

Table 2
Differences in annual spending from perfect certainty

λ	-0.5	-1	-1.5	-2
Scenario 1.1				
c_0	-105	-139	-173	-207
c_1	122	162	201	240
c_2	122	162	201	240
Scenario 1.2				
c_0	-88	-117	-144	-170
c_1	-92	-123	-154	-184
c_2	455	603	747	886
Scenario 2.1				
c_0	-416	-548	-677	-798
c_1	484	639	787	927
c_2	484	639	787	927
Scenario 2.2				
c_0	-334	-428	-511	-584
c_1	-393	-562	-648	-758
c_2	1802	2344	2838	3274
Scenario 3.1				
c_0	-193	-255	-315	-480
c_1	28	36	43	58
c_2	577	766	949	1461
Scenario 3.2				
c_0	-504	-662	-812	-1198
c_1	388	506	616	881
c_2	943	1246	1537	2316
Scenario 3.3				
c_0	-438	-563	-676	-944
c_1	-276	-373	-463	-667
c_2	1927	2511	3042	4285

Note: All consumption levels are in deviations from the certainty equivalent level of 34,305 (the negative of precautionary savings).

Scenarios 1.1 and 1.2 in Table 2 show the effect of timing with moderate uncertainty. When the uncertainty is in the middle period then this leads to more precautionary savings than when the uncertainty is in the distant period. If the elasticity of substitution falls, that is as $-\lambda$ rises, initial period consumption falls by a greater amount (in comparison to the certainty equivalent level). This feature is common to all scenarios. Moreover, as $-\lambda$ rises, virtually all effects are accentuated (across all tables). This is a natural feature of lower elasticity of substitution values. Scenarios 2.1 and 2.2 display the same qualitative patterns but with much greater precautionary savings.[18]

With scenarios 3.1 and 3.2 we see that as Σ_1 rises initial period consumption falls but consumption growth rises. A comparison of scenarios 3.1 and 3.3 shows that increasing $\Sigma_{2|1}$ lowers initial consumption, as with increases in Σ_1, but consumption growth falls. It highlights the role played by more distant income risk in consumption growth. It is interesting to point out how increasing distant uncertainty both reduces near term consumption growth ($c_1 - c_0$) and limits the impact of near term risk. For instance, by comparing scenarios 3.1 and 3.3, we see how consumption growth falls as $\Sigma_{2|1}$ is increased. A comparison of scenarios 3.2 and 3.3 shows that shifting the spread of risk toward the distant future reduces current consumption growth. This is accentuated for less wealthy consumers as we will see in the next section.

4.2. *Effects of wealth*

The relevant impact of income risk typically varies with the level of overall wealth. This is reflected in the CRRA solutions by the presence of terms reflecting the variance of income relative to wealth. In particular, these formulae reflect the sensible course, namely that smaller precautionary savings levels occur for a given amount of income risk at higher wealth levels. Of course, CARA preferences give different results, where precautionary savings are independent of wealth levels.

Table 3 presents optimal savings plans for households with different levels of wealth. In particular, we consider scaled versions of the income profile used above, with 'Low', 'Medium' and 'High' wealth levels corresponding to scale factors of 0.5, 1 and 2.[19] We illustrate the reactions to risk in the 'severe mid-career uncertainty' and 'severe retirement uncertainty' scenarios above. We use CRRA preferences with $\lambda = -1.5$, or a elasticity of substitution of 2/3. We include results for CARA preferences (see Eq. (2.13)) with $\theta = 0.0014$, which we chose so that the CARA results were similar to the CRRA results for the medium wealth scenario.

[18] The innovations being assumed for Scenario 2 are quite large. For instance, Scenario 2.2 could involve a significant probability of zero income in the period 2 (e.g. with truncated normal innovations). We have implicitly assumed that period 2 wealth is bounded away from zero and therefore the possibilty of negative period 2 income is adequately accounted for in the precautionary savings plan.

[19] The fact that we are using scaled versions of the same income profile is not important; in particular, for any given level of wealth, the same precautionary savings would result whether the income stream were constant or varying as in our scenarios. Nonprecautionary savings are fully captured through perfect certainty solutions for consumption.

Table 3
Wealth scenarios

	CARA All wealth values	CRRA Low wealth	Medium wealth	High wealth
Scenario 3.2				
c_0	– 815	– 1449	– 812	– 419
c_1	635	1013	616	325
c_2	1510	2895	1537	782
Scenario 3.3				
c_0	– 744	– 1030	– 676	– 373
c_1	– 382	– 1037	– 463	– 209
c_2	3119	5234	3042	1595

Note: Perfect certainty consumption values are 17,152 (low wealth), 34,305 (medium wealth) and 68,610 (high wealth). All values are differences from perfect certainty consumption expenditures.

The impacts are predictable. In particular, in both scenarios, precautionary savings decline for higher wealth levels under CRRA preferences, but of course are unaffected with CARA preferences. In the case of scenario 3.3 with severe retirement uncertainty, the high retirement income risk keeps consumption low in the first two periods, and especially so for low-wealth households. For low-wealth consumers severe distant income risk is sufficient to induce negative consumption growth even where there is substantial near term income risk. This highlights the disadvantage with CARA preferences, which are unable to detect a drop in consumption growth between the first two periods for the low wealth levels. It is important to again stress that the reported c_1 values are based on a zero income innovation – a nonzero innovation would change wealth and increase the difference between CRRA and CARA solutions.

4.3. Effects of interconnections in income revelations

We now focus on the differences that arise between the case when the revelation of mid-career income is connected to retirement income, and the case when it is not. For this case, we alter the base scenario to have expected income in each period of \$30,000 as of period 0, and we examine only the results of logarithmic CRRA preferences. We consider two situations of unfolding income uncertainty:

No interconnection: In period 1, the consumer has a 50% chance of an income of \$15,000, and a 50% chance of \$45,000, and in period 2, he has a 25% chance of an income of \$5,000, \$25,000, \$35,000 or \$55,000. Here $\Sigma_1 = \$15,000$, $\Sigma_{2|1} = \$18,028$, corresponding to $\sigma_1^2 = 0.022147$ and $\sigma_{2|1}^2 = 0.009807$.

Table 4
Income interconnections

	Perfect certainity	No interconnection	Difference	Full interconnection	Difference
c_0	30000	27553	– 2447	26510	– 3490
c_1	30000	32760	2760	29100	– 900
c_2	30000	36400	6400	39608	9608

Note: 'Difference' refers to the difference from the perfect certainty values.

Full interconnection: In period 1, the consumer has a 50% chance of an income of \$15,000, and a 50% chance of \$45,000, and in period 2, he has a 50% chance of \$10,000 more than his realized period 1 income, and a 50% chance of \$10,000 less. Here $\Sigma_1 = \$23{,}305$, $\Sigma_{2|1} = \$10{,}000$, corresponding to $\sigma_1^2 = 0.053461$ and $\sigma_{2|1}^2 = 0.003017$ (in period 0).

The results of consumption planning under these two scenarios is given in Table 4. In particular, Table 4 gives the consumption plans dictated by the variance of the uncorrelated wealth increments given above. Clearly, with interconnections, a huge amount of the risk is resolved in period 1, and so a greater amount of precautionary savings is done in period 0. Without interconnections, the precautionary savings extend into period 1, to hedge against the large spread of possible incomes in period 2.

This example also provides a rather extreme illustration of a feature of all of the tables presented above; namely that they give savings plans from the vantage point of period 0. As one moves to period 1, the first income innovation is revealed, which would change the period 1 consumption values given above. Likewise, as one moves to period 2, the second innovation is revealed, resulting in the final period 2 consumption value. At any rate, the plans from period 0 may not well resemble savings from a particular household, but more likely reflect an average across households facing similar (but independent) stochastic income environments.

To illustrate this here, we include Table 5, which depicts consumption values in periods 1 and 2 given the realization of period 1's income. As our example has a quite large income change in period 1, quite different consumption values are evidenced. Moreover, the consumption values are quite different depending on whether the large change translates to period 2 (interconnections) or does not (no interconnections).

4.4. The accuracy of the approximations

The above tables are based on our approximation, and so it is important to indicate how accurate our basic approximation is. We first consider the results

Table 5
Evolution of consumption with interconnection

	No interconnection Draw 15000	Full interconnection	No interconnection Draw 45000	Full interconnection
Expected wealth	720,599	592,160.1	1,320,599	1,524,366
c_1	20,546.11	18,330.91	37,653.64	47,188.27
c_2	27,965.543	20,367.68	51,250.78	52,431.41
$c_1 - c_0$	$-$ 7006.82	$-$ 8179.34	10,100.71	20,678.02

presented in Table 2. To compute exact consumption values, we assume that income innovations are drawn from a 95% trimmed normal distribution (the density is set to 0 outside the standard 5% critical values), which has been rescaled to give the variances assumed in Scenarios 1–3. Exact solutions for c_0 are computed numerically, and compared to our approximate c_0 values in Table 6.

We note that our approximation gives smaller values for precautionary savings in all cases, but otherwise are quite close. With moderate uncertainty, approximate precautionary savings are within 5% of exact values. Situations of distant income risk involve larger approximation errors, which measure the impact of compounding errors due to sequential application of the approximation. However, with mixed uncertainty (Scenario 3.1–3.3), the approximation is not worse than with uncertainty in one period only. It should be stressed that we have reported the percentage error in precautionary savings and not in consumption level. For an example, consider the largest error reported, namely with $\lambda = -2$ in Scenario 2.2. Here the approximation error is $(-584 - -745) = 161$ and the (exact) optimal c_0 level is $34{,}305 - 745 = 33{,}560$. Thus the percentage error in precautionary saving is $161/745 = 21.61\%$, but the percentage error in the approximated level of c_0 is $161/33{,}560 = 0.47\%$. In any case, we view all our approximations as quite close for all values in Table 2.

In some ways, the framework assumed for the analysis of income interconnections in Section 4.3 may be the least conducive to the accuracy of our approximation, since the income innovations are assumed there to have discrete distributions. We compute exact values, corresponding to Tables 4 and 5, and present them in Table 7.

In Table 7 the approximate and exact solutions for c_0 and c_1 are presented. In the case of c_1 solutions under each draw, as in Table 5, are reported. The full interconnections case involves a much greater resolution of uncertainty in period 1 and we see a smaller approximation error there. It is evident that the approximation developed in this paper gives the correct intuition in these two

Table 6
Exact solutions and approximation errors for Table 2

λ	− 0.5	− 1	− 1.5	− 2
Scenario 1.1				
c_0	− 105	− 140	− 175	− 213
% Error	0.00	0.71	1.14	2.82
Scenario 1.2				
c_0	− 90	− 121	− 151	− 178
% Error	2.22	3.31	4.64	4.56
Scenario 2.1				
c_0	− 430	− 576	− 717	− 870
% Error	3.26	4.86	5.58	8.28
Scenario 2.2				
c_0	− 374	− 500	− 631	− 745
% Error	10.70	14.4	19.02	21.61
Scenario 3.1				
c_0	− 198	− 265	− 333	− 397
% Error	2.53	3.77	5.41	6.30
Scenario 3.2				
c_0	− 534	− 712	− 889	− 1072
% Error	5.62	7.02	8.66	11.19
Scenario 3.3				
c_0	− 494	− 654	− 818	− 987
% Error	11.34	13.91	17.15	21.27

Note: Consumption levels are in deviations from the certainty equivalent level of 34,305. Percentage error refers to the percentage difference between the approximate and exact precautionary savings values.

Table 7
Exact solutions with income interconnections

	No interconnection	Full interconnection	No interconnection	Full interconnection
c_0 Approx.	27,553	26,310		
c_0 Exact	26,525	26,158		
% Error	30.00	3.96		
	Draw 15000		Draw 45000	
c_1 Approx.	20,546	18,330	37,654	47,188
c_1 Exact	19,600	17,872	40,986	48,764

scenarios. The jump up in c_1 for the good draw and the corresponding jump down for the bad draw are well tracked by the approximate solutions.

5. Summary and conclusions

This article has examined the impact of income uncertainty on optimal consumption expenditures, when the uncertainty in income is either near-term or in the more distant future. Using a three-period framework, we have given approximate solutions for optimal consumption choices for several standard preference types; quadratic, preferences that display constant absolute risk aversion and preferences that display constant relative risk aversion. These solutions are designed to provide good intuition without recourse to dynamic programming solutions.

The importance of near-term versus more distant risk in the analysis of consumption growth is evaluated and solutions are provided for each preference type. We relate income innovations to consumption innovations and detail how the timing of risk effects consumption growth. In each area we highlight the differences in behavior implied by the different preference specifications.

One important lesson from this work is how the entire structure of risk aversion in preferences is relevant for understanding the impact of income risk. Our different preference types give entirely different reactions to income risk at different points in the future. We regard the preferences of the CRRA class as perhaps the most realistic for modeling actual savings behavior in empirical work, because they can capture the most plausible precautionary behavior for rich and poor households. Our view is that quadratic preferences, which are risk neutral, or CARA preferences, for which precautionary behavior is independent of wealth, each contain serious handicaps for the purpose of capturing precautionary savings. However, the main point is that the depiction of precautionary behavior is particularly sensitive to the choice of preferences, and so deserves explicit attention in model design.

With regard to CRRA preferences, we have noted the importance of the level of wealth, the timing of income risk, and the elasticity of intertemporal substitution. With the three-period framework, we have developed formulae that relate these dimensions to consumption levels and consumption growth, and we feel these formulae can be useful for empirical work. The timing of income risk is shown to be critical. Income uncertainty alone is not sufficient to induce positive consumption growth. What matters is precisely when the income uncertainty is resolved. In some cases, even with near term risk, a relatively high degree of distant risk can induce small or even negative consumption growth. This is especially pronounced for poorer, more risk averse, consumers.

As with the related literature, we have utilized approximations to solve for consumption values with CRRA preferences. However, our approximations are novel to the extent that they permit multiperiod intertemporal allocation solutions to be derived, in that future consumption decisions are easily taken into account in current consumption decisions. In particular, we use adjusted wealth shares and approximations to the first-order conditions based on expanding around the solutions for perfect certainty which will be particularly accurate when the innovations to wealth are small. We provide a comparison with exact solutions for some our some income risk scenarios and find that the approximations closely track the full solution even where there are interconnections in income risk across periods.

This framework would easily extend to solutions of models of more than three periods, which would be called for in a more detailed setting for household savings; say early worklife aimed at home purchase, raising of children, college expenditures, retirement costs and then bequests. As such, we see our focus on three periods as having two valuable aspects: first as indicating a method of giving useful multiperiod (approximate) solutions, and second as revealing the limitations of the standard two-period analysis.

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Appendix A: Quadratic and CARA preferences

A.1. Quadratic preferences

Consider first the case of quadratic preferences (see Hall, 1978). Here the felicity function is

$$U_t(c_t) = -(\alpha_t - c_t)^2/2$$

where α_t reflects planned variations in expenditures over the three periods, and $c_t \leq \alpha_t$ is assumed. Solving Problem 1 (i.e. Eq. (2.10)) gives

$$c_1 = \frac{\phi_2\alpha_1 - \alpha_2}{1 + r_2 + \phi_2} + \frac{1 + r_2}{1 + r_2 + \phi_2} W_1 \tag{A.1}$$

where $\phi_2 = (1+\delta_2)/(1+r_2)$. The solution for c_0 takes the same general linear form as for c_1 but this time it is linear in period 0 expected wealth W. The implied growth of consumption is given as

$$c_{t+1} = \phi_{t+1}c_t + (\alpha_{t+1} - \phi_{t+1}\alpha_t) + u_{t+1} \tag{A.2}$$

where $u_{t+1} = (\alpha_t - c_t)\varepsilon_{t+1}$ and $\mathrm{E}_t(u_{t+1}) = 0$. Income and consumption innovations are easily connected; namely $u_1 = \omega\zeta_1^*$ with $\omega = (1+r_2)/(1+r_2+\phi_2)$, see Blundell and Stoker (1997).

The solutions under quadratic preferences obey certainty equivalence: c_0 matches the perfect certainty solution c_0^{o}, and c_1 is the perfect certainty value with wealth updated for the revelation of ζ_1^*; namely

$$c_1 = c_1^{\mathrm{o}} + \frac{1+r_2}{1+r_2+\phi_2}\zeta_1^*.$$

In terms of the original innovation ζ_1, with cumulative adjustment (2.5), we have

$$c_1 = c_1^{\mathrm{o}} + \frac{1+r_2}{1+r_2+\phi_2}\left[\zeta_1 + \frac{\mathrm{E}_1(\zeta_2 \mid \zeta_1)}{1+r_2}\right] = c_1^{\mathrm{o}} + \left[1 + \frac{b-\phi_2}{1+r_2+\phi_2}\right]\zeta_1.$$

With ζ_1 revealed, wealth is updated by ζ_1^* and consumption is adjusted for the new wealth level as above. In brief, for quadratic preferences, income risk plays no role; only the means of unknown income innovations enter optimal consumption values.

A.2. CARA preferences

An explicit solution is also available with CARA preferences, which are characterized by an exponential felicity function of the form

$$U_t(c_t) = \mathrm{e}^{\theta(\alpha t - ct)}/-\theta,$$

where again α_t reflects planned variations in expenditures over time, and θ reflects the degree of risk aversion. Here, the first-order condition (2.10) for Problem 1 becomes

$$\mathrm{e}^{\theta(\alpha 2-(1+r_2)(W_1-c_1))}\mathrm{E}_1[\mathrm{e}^{-\theta\zeta_2^*}] = \phi_2\mathrm{e}^{\theta(\alpha_1-c_1)}. \tag{A.3}$$

The solution for c_1 takes the form

$$c_1 = K_1 + K_2m_2(\zeta_1) + K_3W_1 \tag{A.4}$$

where K_1, K_2 and K_3 are functions of α_1, α_2, θ and r_2, with income risk enters only through the term $m_2(\zeta_1) \equiv \ln \mathrm{E}_1(\mathrm{e}^{-\theta\zeta_2^*}|\zeta_1)$. A more interpretable form of the risk effect arises from using the first-order approximation

$m_2(\zeta_1) \cong (\theta^2/2)\Sigma^2_{2|1}$;[20] for instance, c_1 is decreased as the conditional variance $\Sigma^2_{2|1}$ of ζ^*_2 is increased (since $K_2 < 0$). CARA preferences likewise imply the same kind of structure for Problem 0; namely

$$c_0 = k_0 + k_1 m_1 + k_2 m_2 + k_3 W \tag{A.5}$$

where m_1 and m_2 are analogous income risk terms, and k_0, k_1, k_2 and k_3 are functions of preference and other parameters (see Blundell and Stoker, 1997).

The same features are exhibited by consumption growth equations. Here the stochastic Euler condition can be written

$$c_{t+1} - c_t = \left(\frac{1}{\theta}\right) \ln \phi_{t+1} + (\alpha_{t+1} - \alpha_t) + \left(\frac{1}{\theta}\right) m_{t+1} + \left(\frac{1}{\theta}\right) \tilde{u}_{t+1} \tag{A.6}$$

in which $\mathrm{E}_t(\tilde{u}_{t+1}) = 0$. Using Eq. (2.4), consumption innovations and income innovations are related as

$$\tilde{u}_2 = \theta \zeta^*_2 \quad \text{and} \quad \tilde{u}_1 = \theta \frac{(1 + r_2)}{(2 + r_2)} \zeta^*_1. \tag{A.7}$$

Again, the adjustments for risk are linear, and independent of the level of wealth. Consequently, while CARA preferences do permit explicit solutions for intertemporal allocation to be derived, the solutions do not represent rich–poor planning distinctions very realistically.

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